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Mirage cosmology

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Abstract: A brane universe moving in a curved higher dimensional bulk space is considered. The motion induces a cosmological evolution on the universe brane that is indistinguishable from a similar one induced by matter density on the brane. The phenomenological implications of such an idea are discussed. Various mirage energy densities are found, corresponding to dilute matter driving the cosmological expansion, many having superluminal properties $|w| > 1$ or violating the positive energy condition. It is shown that energy density due to the world-volume fields is nicely incorporated into the picture. It is also pointed out that the initial singularity problem is naturally resolved in this context.

Keywords: Superstrings and Heterotic Strings, D-branes, Classical Theories of Gravity, Cosmology of Theories beyond the SM.

Contents

1. Introduction

In this paper we will elaborate on some ideas related to the possibility that our observable four-dimensional world is a three-brane embedded in ten dimensional string theory. There has been earlier speculations [1] stating that our observable four-dimensional universe is a domain wall embedded in a higher dimensional space. Although there is no strong theoretical or experimental motivation for this idea, it has been revived recently [2]–[11] motivated by the possibility of large compact internal dimensions [12] the notion and existence of D-branes in string theory [13], and the fact that orientifold [14] and D-manifold [15] vacua of string theory can be thought of as lower-dimensional D-branes embedded in a ten-dimensional bulk. An early example of this is the Hořava-Witten picture for the non-perturbative heterotic $E_8 \times E_8$ string [16], the relevance of this for gauge coupling unification upon compactification to five dimensions [17] and the associated picture of supersymmetry breaking [18].

An extra motivation is the will of theorists to provide with potential new physics signals the experimental physics community. Here, in particular, the new physics can be string effects, or quantum gravitational effects at scales that are well below the four-dimensional Planck scale. One can avoid typical string theory constraints by focusing on type-I (orientifold) vacua of string theory. Although this approach has not obviously solved any major theoretical problem yet it is an interesting alternative and its implications should be pursued.

In the context we will assume, the Standard Model gauge bosons as well as charged matter arise as fluctuations of the D-branes. We can thus consider the universe (standard model) to be living on a collection of coincident branes, while hidden gauge interactions can be localized on other branes. Gravity as well as other universal interactions is living in the bulk space.

There is an approximation which is very useful in order to treat the dynamics of the universe brane. This is the probe limit in which the influence of the probe brane source to the bulk fields is negligible. This has been a natural and useful tool [20]–[22] in order to understand issues in the context of AdS/CFT correspondence [23]. However, for the spherically symmetric bulk configurations we will consider, the probe limit will give exact results.

There has been a lot of recent work on the potential cosmological models associated to a brane universe [24]–[39]. Our approach will be slightly different. We can imagine the collection of other branes to provide a gravitational background which is felt by the universe brane treated as a probe. In this context a universe three-brane (or higher compactified brane) can be in motion in ten-dimensional space in the presence of a the gravitational field of the other branes. We ignore its back reaction to the ambient geometry. We will show that the motion in ambient space induces cosmological expansion (or contraction) on our universe simulating various kinds of "matter" or a cosmological constant (inflation). This is what we mean by mirage cosmology: the cosmological expansion is not due to energy density on our universe but somewhere else. This can be either on other branes (that can be represented qualitatively by a black hole background) or in the bulk.

There is a different limit in which our universe is moving to a region of ultra-weak bulk fields, in which case the matter density on it alone drives the cosmological expansion, in the traditional fashion. We will show that this limit is equally well described in our setup.

The holographic principle and AdS/CFT correspondence ideas, are providing novel ways to treat old mechanisms. In the case of cosmological expansion, supersymmetry is very softly broken and it is expected that the probe approximation may be valid even in the case where the source is not hierarchically heavier than the probe [22]. It was observed that for a D-brane moving in the background of a black D-brane the word-volume theory has an effective speed of light which is field dependent [22]. Once the probe brane is in geodesic motion the varying speed of light is equivalent to cosmological expansion on the probe brane.

There are two possibilities to be explored in relation with the bulk geometry. The bulk may not be compact (but there is a mass gap [40] or some way it makes low lying higher-dimensional gravitons unobservable [6]). Then, there is no bound on the mass of D-branes. If the bulk is compact there are charge neutrality constraints that must be satisfied and they constrain the brane configurations. In simple situations they limit the number of D-branes. A typical example is the D9-branes in type-I string theory whose number is limited to 32.

Here we will mostly focus in the non-compact case. In the case where the space is compact the generalization (and limitation) of our arguments will be straightforward. For regions which are small compared to the size of the compact space the description of geodesics is accurate. When a geodesic reaches distances comparable with the compact size we must use the form of bulk solution which is periodic (and can be constructed as an infinite periodic array of non-compact solutions). Using such a matching formula the full geodesics can be studied. This will imply that in such a context the most probable cosmological evolution is a bouncing one.

Thus, the central idea is that the universe brane is moving into the bulk background fields of other branes of the theory. The motion of the brane follows thus, a classical geodesic in the bulk geometry. The prototype branes we are using here are Dp-branes with maximal supersymmetry. Any realistic type-I ground-state can be viewed in big region of moduli space as intersecting such branes. Moreover we know well the coupling of world-volume fields to the bulk supergravity fields.

There are two steps in the procedure:

- Determine the brane motion by solving the world-volume field equations for the scalar fields determining the position of the brane in the bulk

- Determine the induced metric on the brane which now becomes an implicit function of time. This gives a cosmological evolution in the induced brane metric. This cosmological evolution can be reinterpreted in terms of cosmological "mirage" energy densities on the brane via a Friedman-like equation. The induced metric on the brane is the natural metric felt by the observers on the brane. We assume that our universe lives on the brane and is made off open string fluctuations.

- Similarly one can determine the other interactions on the brane

An important reminder here is that the cosmological evolution is *not* driven by four-dimensional gravity on the brane. Our analysis indicates that potential inflationary models where the position scalar and its potential is used to generate inflation on the brane via conventional four-dimensional gravity couplings might not after all generate the sought-after inflation. A necessary condition is that the minimum of the potential (equal to the brane tension) to be above the BPS limit.

We have analysed various combinations of branes and simple background fields. They correspond to a stack of Dp-branes on and out of extremality (black Dp-branes) as well as with additional constant antisymmetric tensor backgrounds. They provide a cosmological evolution on the probe brane that can be simulated by various types

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of mirage matter on the brane. Most prominent is radiation-types ($w = 1/3$) or massless scalars ($w = 1$). It should be stressed however that at small scale factor size, there are many exotic types of mirage matter including w values that are outside the range $|w| \leq 1$ required by four-dimensional causality. We interpret this as an indication that superluminal (from that four-dimensional points of view) "shocks" are possible in such cosmologies. Superluminal signal propagation in a brane-world context have been recently pursued independently in [39, 41].

Another peculiarity is that individual densities of mirage dilute matter can be negative (without spoiling the overall positivity at late times). We think that this is linked to the fact that in this type of cosmology the initial singularity is an artifact of the low energy description.

This can be seen by studying brane motion in simple spaces like $AdS_5 \times S^5$ which are globally non-singular. The induced cosmological evolution of a brane moving in such a space has a typical expansion profile due to radiation and an initial singularity (from the four-dimensional point of view). However this singularity is an artifact. At the point of the initial singularity the universe brane joins a collection of parallel similar branes and there is (non-abelian) symmetry enhancement. The effective field theory breaks down and this gives rise to the singularity.

The next obvious question is how "real" matter/energy densities on the brane affect its geodesic motion and consequently the induced cosmological evolution. This can be studied by turning on electromagnetic energy on the brane. We find a solution of the moving brane with a covariantly constant electric field. We do show that this gives an additional effect on the cosmological evolution similar to the analogous problem of radiation density in four-dimensions. Although an electric field is an unrealistic cosmological background the solution we obtain is valid when the electric energy density is thermal (and thus isotropic) in nature. This indicates that the formalism we present is capable of handling the most general situation possible, namely cosmological evolution driven by bulk background fields (mirage matter) as well as world-volume energy densities (real matter).

As it was first pointed out in [22] this context allows for an arbitrarily small residual cosmological constant on the universe brane.

The structure of the paper is as follows. In section 2 we develop the formalism for D-brane geodesics in non-trivial bulk metric as well as RR form. The induced cosmological evolution is also derived. In section 3 we study the influence on the geodesics of an additional constant NS antisymmetric tensor background. In section 4 we derive the geodesic equations in the presence of an electric field background on the probe brane. In section 5 we study the Friedman-like equations for various examples of bulk fields. In section 6 we discuss the nature and fate of the initial (cosmological) singularity. In section 7 we briefly discuss how the cosmological setup presented can be incorporated in string theory. Finally section 8 contains our conclusions and open problems.

2. Brane geodesics

In this section we will consider a probe D-brane moving in a generic static, spherically symmetric background. The brane will move along a geodesic. We assume the brane to be light compared to the background so that we will neglect the back-reaction. The idea here is that as the brane moves along a geodesic, the induced world-volume metric becomes a function of time, so that from the brane "residents" point of view they are living in a changing (expanding or contracting) universe.

The simplest case corresponds to a D3-brane and we will focus mostly on this case. Later we will consider Dp-branes with $p > 3$ and $p-3$ coordinates compactified. The metric of a D3-brane may be parametrized as

$$ds_{10}^2 = g_{00}(r)dt^2 + g(r)(d\vec{x})^2 + g_{rr}(r)dr^2 + g_S(r)d\Omega_5 \tag{2.1}$$

and we may also generically have a dilaton $\phi(r)$ as well as a RR background $C(r) = C_{0\ldots3}(r)$ with a self-dual field strength. The probe brane will in general start moving in this background along a geodesic and its dynamics is governed by the DBI action. In the case of maximal supersymmetry it is given by

$$S = T_3 \int d^4\xi e^{-\phi}\sqrt{-\det(\hat{G}_{\alpha\beta} + (2\pi\alpha')F_{\alpha\beta} - B_{\alpha\beta})} + T_3 \int d^4\xi\, \hat{C}_4 + \text{anomaly terms}\,, \tag{2.2}$$

where we have ignored the world-volume fermions. The emmbeded data are given by

$$\hat{G}_{\alpha\beta} = G_{\mu\nu}\frac{\partial x^\mu}{\partial\xi^\alpha}\frac{\partial x^\nu}{\partial\xi^\beta}\,, \text{ etc.} \tag{2.3}$$

Due to reparametrization invariance, there is a gauge freedom which may be fixed by choosing the static gauge, $x^\alpha = \xi^\alpha$, $\alpha = 0, 1, 2, 3$. A generic motion of the probe D3-brane will have a non-trivial angular momentum in the transverse directions. In the static gauge the relevant (bosonic) part of the brane lagrangean reads

$$L = \sqrt{g(r)^3[|g_{00}| - g_{rr}\dot{r}^2 - g_S(r)h_{ij}\dot{\varphi}^i\dot{\varphi}^j]} - C(r)\,, \tag{2.4}$$

where $h_{ij}(\varphi)d\varphi^i\varphi^j$ is the line element of the unit five-sphere. For future purposes (generality) we will parametrize the lagrangean as

$$\mathcal{L} = \sqrt{A(r) - B(r)\dot{r}^2 - D(r)h_{ij}\dot{\varphi}^i\dot{\varphi}^j} - C(r)\,, \tag{2.5}$$

with

$$A(r) = g^3(r)|g_{00}(r)|e^{-2\phi}\,, \qquad B(r) = g^3(r)g_{rr}(r)e^{-2\phi}\,, \qquad D(r) = g^3(r)g_S(r)e^{-2\phi} \tag{2.6}$$

and $C(r)$ is the RR background. The problem is effectively one-dimensional and can be solved by quadratures. The momenta are given by

$$p_r = -\frac{B(r)\dot{r}}{\sqrt{A(r) - B(r)\dot{r}^2 - D(r)h_{ij}\dot{\varphi}^i\dot{\varphi}^j}}\,,$$
$$p_i = -\frac{D(r)h_{ij}\dot{\varphi}^j}{\sqrt{A(r) - B(r)\dot{r}^2 - D(r)h_{ij}\dot{\varphi}^i\dot{\varphi}^j}}\,. \tag{2.7}$$

The angular momenta as well as the hamiltonian

$$H = -E = C - \frac{A(r)}{\sqrt{A(r) - B(r)\dot{r}^2 - D(r)h_{ij}\dot{\varphi}^i\dot{\varphi}^j}} \tag{2.8}$$

are conserved. The conserved total angular momentum (SO(6) quadratic Casimir in our case) is $h^{ij}p_ip_j = \ell^2$ and

$$h_{ij}\dot{\varphi}^i\dot{\varphi}^j = \frac{\ell^2(A(r) - B(r)\dot{r}^2)}{D(r)(D(r)+\ell^2)}\,. \tag{2.9}$$

Thus, the final equation for the radial variable is

$$\sqrt{\frac{D}{D+\ell^2}(A(r) - B(r)\dot{r}^2)} = \frac{A(r)}{E + C(r)}\,. \tag{2.10}$$

In summary,

$$\dot{r}^2 = \frac{A}{B}\left(1 - \frac{A}{(C+E)^2}\frac{D+\ell^2}{D}\right), \qquad h_{ij}\dot{\varphi}^i\dot{\varphi}^j = \frac{A^2\ell^2}{D^2(C+E)^2}\,. \tag{2.11}$$

Here we see that we have the constraint that $C(r) + E \geq 0$ for the allowed values of r. A stronger condition can be obtained from (2.11)

$$\frac{A}{B}\left(1 - \frac{A}{(C+E)^2}\frac{D+\ell^2}{D}\right) \geq 0\,. \tag{2.12}$$

Note that for slow motion (non-relativistic limit), we can expand the square root in (2.5) to obtain

$$\mathcal{L}_{\text{n.r.}} = \sqrt{A(r)} - C(r) - \frac{1}{2}\frac{B(r)}{\sqrt{A(r)}}\dot{r}^2 - \frac{1}{2}\frac{D(r)}{\sqrt{A(r)}}h_{ij}\dot{\varphi}^i\dot{\varphi}^j\,, \tag{2.13}$$

which is equivalent to a particle moving in a potential of a central force. The analogous solution is now

$$\dot{r}^2 = 2\left[E + C - \sqrt{A}\right]\frac{\sqrt{A}}{B} - \frac{A\ell^2}{BD}\,. \tag{2.14}$$

The non-relativistic limit is valid when $A/(C+E)^2 \simeq 1$ and $D(r) \gg \ell^2$.

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The induced four-dimensional metric on the 3-brane universe is

$$d\hat{s}^2 = \left(g_{00} + g_{rr}\dot{r}^2 + g_S h_{ij}\dot{\varphi}^i\dot{\varphi}^j\right) dt^2 + g(d\vec{x})^2 \tag{2.15}$$

and upon substituting from (2.6) it becomes

$$d\hat{s}^2 = -\frac{g_{00}^2 g^3 e^{-2\phi}}{(C+E)^2} dt^2 + g(d\vec{x})^2 \,. \tag{2.16}$$

We can define the cosmic time η as

$$d\eta = \frac{|g_{00}| g^{3/2} e^{-\phi}}{|C+E|} dt \,, \tag{2.17}$$

so that the universe metric is

$$d\hat{s}^2 = -d\eta^2 + g(r(\eta))(d\vec{x})^2 \,. \tag{2.18}$$

The cosmic time is the same as the proper time of the universe brane. Equation (2.18) is the standard form of a flat expanding universe. We can now derive the analogues of the four-dimensional Friedman equations by defining the scale factor as $a^2 = g$. Then,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{(C+E)^2 g_S e^{2\phi} - |g_{00}|(g_S g^3 + \ell^2 e^{2\phi})}{4|g_{00}| g_{rr} g_S g^3} \left(\frac{g'}{g}\right)^2 , \tag{2.19}$$

where the dot stands for derivative with respect to cosmic time while the prime stands for derivative with respect to r. The right hand side of (2.19) can be interpreted in terms of an effective matter density on the probe brane

$$\frac{8\pi}{3}\rho_{\text{eff}} = \frac{(C+E)^2 g_S e^{2\phi} - |g_{00}|(g_S g^3 + \ell^2 e^{2\phi})}{4|g_{00}| g_{rr} g_S g^3} \left(\frac{g'}{g}\right)^2 . \tag{2.20}$$

We have also

$$\frac{\ddot{a}}{a} = \left(1 + \frac{g}{g'}\frac{\partial}{\partial r}\right) \frac{(C+E)^2 g_S e^{2\phi} - |g_{00}|(g_S g^3 + \ell^2 e^{2\phi})}{4|g_{00}| g_{rr} g_S g^3} \left(\frac{g'}{g}\right)^2 = \left[1 + \frac{1}{2}a\frac{\partial}{\partial a}\right] \frac{8\pi}{3}\rho_{\text{eff}} \,. \tag{2.21}$$

By setting the above equal to $-\frac{4\pi}{3}(\rho_{\text{eff}} + 3p_{\text{eff}})$ we can define the effective pressure p_{eff}.

In terms of the above we can calculate the apparent scalar curvature of the four-dimensional universe as

$$R_{4-d} = 8\pi(\rho_{\text{eff}} - 3p_{\text{eff}}) = 8\pi\left(4 + a\partial_a\right)\rho_{\text{eff}} \,. \tag{2.22}$$

In the non-relativistic limit the induced metric is approximated by

$$d\hat{s}^2\Big|_{n.r.} \simeq g_{00}\left[1 - 2\left(\frac{(C+E)e^{\phi}}{\sqrt{g^3|g_{00}|}} - 1\right) - \frac{\ell^2 e^{2\phi}}{g^3 g_S}\right] dt^2 + g(d\vec{x})^2 \,, \tag{2.23}$$

while the effective density becomes

$$\frac{8\pi}{3}\rho_{\text{eff}}\Big|_{n.r} \simeq \frac{1}{4g_{rr}}\left(\frac{g'}{g}\right)^2\left[\left(\frac{(C+E)e^{\phi}}{\sqrt{g^3|g_{00}|}}-1\right)-\frac{\ell^2 e^{2\phi}}{g^3 g_S}\right]. \tag{2.24}$$

As we will see in specific examples later on, this approximation is usually valid for motion far away from the source corresponding to large (late) time of the scale factor. Moreover, going back towards what is the initial singularity, relativistic corrections become increasingly important.

The discussion above may easily be generalized for the geodesic motion of a probe Dp-brane in the field of a Dp'-brane with $p' > p$. In this case, the Dp'-brane metric is of the form

$$ds_{10}^2 = g_{00}(r)dt^2 + g(r)(d\vec{x}_{p'})^2 + g_{rr}(r)dr^2 + g_S(r)d\Omega_{8-p'}\,, \tag{2.25}$$

and there exist in general a non-trivial dilaton profile $\phi = \phi(r)$ as well as a RR $p'+1$ from $C_{p'+1}$. The Dp-brane probe in this background will feel only gravitational and dilaton forces since it has no p'-brane charge. Its motion will then determined by the DBI action

$$S_p = T_p\int d^{p+1}\xi e^{-\phi}\sqrt{-\det(\hat{G}_{\alpha\beta})}\,. \tag{2.26}$$

In the static gauge and for a generic motion with non-trivial angular-momentum in the transverse directions of the Dp'-brane we find that the lagrangean turns out to be

$$\mathcal{L} = \sqrt{A_p(r) - B_p(r)\dot{r}^2 - D_p(r)h_{ij}\dot{\varphi}^i\dot{\varphi}^j}\,, \tag{2.27}$$

where now

$$\begin{aligned} A_p(r) &= g^p(r)|g_{00}(r)|e^{-2\phi}\,,\\ B_p(r) &= g^p(r)g_{rr}(r)e^{-2\phi}\,,\\ ,\qquad D_p(r) &= g^p(r)g_S(r)e^{-2\phi}\,. \end{aligned} \tag{2.28}$$

Proceeding as before, we find the induced metric on the Dp-brane,

$$d\hat{s}^2 = (g_{00} + g_{rr}\dot{r}^2 + g_S h_{ij}\dot{\varphi}^i\dot{\varphi}^j)dt^2 + g(d\vec{x}_p)^2 \tag{2.29}$$

and upon substitution becomes

$$d\hat{s}^2 = -\frac{g_{00}^2 g^p e^{-2\phi}}{E^2}dt^2 + g(d\vec{x}_p)^2\,. \tag{2.30}$$

We can define now the cosmic time η as

$$d\eta = \frac{|g_{00}|g^{p/2}e^{-\phi}}{|E|}dt\,, \tag{2.31}$$

so that the universe metric is $d\hat{s}^2 = -d\eta^2 + g(r(\eta))(d\vec{x})^2$. The analogues of the $p+1$-dimensional Friedman equations are determined by defining the scale factor as $a^2 = g$. As before we obtain a Friedman type equation with an effective density given by

$$\frac{4\pi}{3}\rho_{\rm eff} = \frac{E^2 g_S e^{2\phi} - |g_{00}|(g_S g^p + \ell^2 e^{2\phi})}{4|g_{00}|g_{rr}g_S g^p}\left(\frac{g'}{g}\right)^2 . \tag{2.32}$$

Note that in the case $p = p'$, there exist the additional WZ term $T_p\int\hat{C}_{p+1}$ in the action (2.26) which modifies the equations of motion of the probe brane as well as the induced metric. This modification is nothing than the shift $E \to E + C$ where $C = C_{0\ldots p}$.

3. Branes with constant B-fields

Interesting dynamics may also arise by turning on the antisymmetric NS/NS field $B = \frac{1}{2}B_{\mu\nu}dx^\mu \wedge dx^\nu$ on the macroscopic Dp-brane. In particular, a constant B will not affect the background since it enters in the field equations via its field strength $H = dB$ which is zero for a constant B. However, a probe brane feels not H but the antisymmetric field itself through the coupling

$$S = T_p\int d^{p+1}\xi e^{-\phi}\sqrt{-\det(\hat{G}_{\alpha\beta} - \hat{B}_{\alpha\beta})} + T_p\int d^{p+1}\xi\hat{C}_{p+1} + \text{anomaly terms}\,, \tag{3.1}$$

where

$$\hat{B}_{\alpha\beta} = B_{\mu\nu}\frac{\partial x^\mu}{\partial\xi^\alpha}\frac{\partial x^\nu}{\partial\xi^\beta}\,. \tag{3.2}$$

We will assume again that there exist a macroscopic Dp'-brane with a background metric as in eq. (2.25) and a probe Dp-brane with $p < p'$. In addition to the dilaton $\phi(r)$ and RR form $C_{p'+1}$, the Dp'-brane now supports a constant NS/NS two-form in its world-volume which we take to be is $B = b dx^{p-1} \wedge dx^p$. In general an antisymmetric tensor in the direction of the universe brane will break isotropy. However, in the case where the universe brane is higher-dimensional with some of the directions compactified (which is the realistic situation in type-I string theory) then we can turn on a component $B_{0\theta}$ where θ is one of the compact directions. Such an antisymmetric tensor does not break isotropy in the effective four-dimensional universe.

In the case at hand the induced antisymmetric two-form on the probe Dp-brane and its motion is determined by the lagrangean

$$\mathcal{L} = \sqrt{K_p(r) - L_p(r)\dot{r}^2 - N_p(r)h_{ij}\dot{\varphi}^i\dot{\varphi}^j}\,, \tag{3.3}$$

where now

$$\begin{aligned}K_p(r) &= g(r)^{p-2}(g(r)^2+b^2)|g_{00}(r)|e^{-2\phi}\,,\\ L_p(r) &= g(r)^{p-2}(g(r)^2+b^2)g_{rr}(r)e^{-2\phi}\,,\\ N_p(r) &= g(r)^{p-2}(g(r)^2+b^2)g_S(r)e^{-2\phi}\,.\end{aligned} \tag{3.4}$$

First integrals of the equations of motion are now

$$\dot{r}^2 = \frac{K_p}{L_p}\left(1-\frac{K_p(N_p+\ell^2)}{N_pE^2}\right), \qquad h_{ij}\dot{\varphi}^i\dot{\varphi}^j = \frac{K_p^2\ell^2}{N_p^2E^2}\,, \tag{3.5}$$

while the induced metric will still be given by eq. (2.15). Proceeding as before we find that the Friedman-like equations are

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{E^2g_Se^{2\phi}-|g_{00}|\left(g_Sg^{p-2}(g^2+b^2)+\ell^2e^{2\phi}\right)}{4|g_{00}|g_{rr}g_Sg^{p-2}(g^2+b^2)}\left(\frac{g'}{g}\right)^2\,. \tag{3.6}$$

In addition we have

$$\frac{\ddot{a}}{a} = \left(1+\frac{g'}{4g}\frac{\partial}{\partial r}\right)\frac{E^2g_Se^{2\phi}-|g_{00}|\left(g_Sg^{p-2}(g^2+b^2)+\ell^2e^{2\phi}\right)}{4|g_{00}|g_{rr}g_Sg^{p-2}(g^2+b^2)}\left(\frac{g'}{g}\right)^2. \tag{3.7}$$

Consequently, the effective energy density on the probe brane turns out to be

$$\frac{4\pi}{3}\rho_{\rm eff} = \frac{E^2g_Se^{2\phi}-|g_{00}|\left(g_Sg^{p-2}(g^2+b^2)+\ell^2e^{2\phi}\right)}{4|g_{00}|g_{rr}g_Sg^{p-2}(g^2+b^2)}\left(\frac{g'}{g}\right)^2\,. \tag{3.8}$$

In the case $p = p'$, the probe brane feels also the RR form so that the effective energy density turns out to be

$$\frac{4\pi}{3}\rho_{\rm eff} = \frac{(C+E)^2g_Se^{2\phi}-|g_{00}|\left(g_Sg^{p-2}(g^2+b^2)+\ell^2e^{2\phi}\right)}{4|g_{00}|g_{rr}g_Sg^{p-2}(g^2+b^2)}\left(\frac{g'}{g}\right)^2\,. \tag{3.9}$$

4. Electric fields on the brane

Let us now assume that we turn on an electric field on the probe D3-brane. This obviously breaks isotropy on the universe and thus this is not a realistic configuration for cosmological purposes. Our aim here is different. We would like to show that when there energy density due to the gauge fields of the brane, our approach takes them appropriately into account and they will affect the evolution of the cosmological factor. Moreover, it is not difficult to argue that if the gauge-field energy density is thermal in nature (and thus not isotropy breaking) this will not affect our conclusions provided we substitute $\vec{E}^2 \to \langle \vec{E}^2 \rangle$ etc. The end result turns out to be that our approach takes into account properly both mirage and real energy densities and one can eventually study the transition between the two.

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When we keep track of the gauge fields, the action for the D3-brane is given in (2.2) and for the background in (2.1) the lagrangean takes the form

$$\mathcal{L} = \sqrt{A - B\dot{r}^2 - \mathcal{E}^2 g^2} - C\,, \tag{4.1}$$

where $\mathcal{E}^2 = 2\pi\alpha' E_i E^i$ and $E_i = -\partial_t A_i(t)$ in the $A_0 = 0$ gauge and A, B are given in (2.6). For simplicity we focus on radial motion.

The equations of motions for the electric field turn out to be

$$\partial_t \left(\frac{g^2 E_i}{\sqrt{A - B\dot{r}^2 - \mathcal{E}^2 g^2}} \right) = 0\,. \tag{4.2}$$

and we find that

$$E_i = \frac{\mu_i}{g}\sqrt{\frac{A - B\dot{r}^2}{\mu^2 + g^2}}\,, \qquad \mathcal{E}^2 = \frac{\mu^2}{g^2}\frac{A - B\dot{r}^2}{g^2 + \mu^2}\,, \tag{4.3}$$

where μ_i is an integration constant and $\mu^2 = (2\pi\alpha')\mu_i\mu^i$. In the case $\dot{r} = 0$, E_i is constant as it is required by ordinary Maxwell equations. A first integral is given by

$$\dot{r}^2 = \frac{A}{B}\left(1 - \frac{A}{(C+E)^2(1+\mu^2 g^{-2})}\right). \tag{4.4}$$

Using (4.4), we obtain

$$\mathcal{E}^2 = \mu^2 \frac{A^2}{(C+E)^2(g^2+\mu^2)^2}\,. \tag{4.5}$$

The induced metric on the probe D3-brane turns out to be

$$d\hat{s}^2 = -\frac{g_{00}^2 g^5 e^{-2\phi}}{(C+E)^2(\mu^2+g^2)}dt^2 + g(d\vec{x})^2 \tag{4.6}$$

and by defining the cosmic time η along similar lines we obtain the following analog of the Friedman equations

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{(C+E)^2(1+\mu^2 g^{-2}) - |g_{00}|g^3 e^{-2\phi})}{4|g_{00}|g_{rr}g^3 e^{-2\phi}}\left(\frac{g'}{g}\right)^2. \tag{4.7}$$

We note that the dominant contribution to $(\dot{a}/a)^2$ from the electric field is of order $\mathcal{E}^2$ as can be seen from eq. (4.5) and thus proportional to the energy density $\rho \sim \mathcal{E}^2$. It should also be noted that there exist a limiting value for the electric field [42, 43] which is

$$\mathcal{E}^2 \le A^2 g^{-2}\,. \tag{4.8}$$

The gauge invariance of the bulk antisymmetric tensor is closely tied with that of the world-volume gauge fields. This is in agreement with the observation that, comparing equations (3.9) (with $\ell = 0$) and (3.6) that an electric field on the brane or a constant antisymmetric tensor in bulk produce similar cosmological evolution.

5. Cosmology of a probe D3-brane in various backgrounds

In this section we will analyse several concrete bulk configurations and elaborate on the induced cosmological expansion on the universe brane.

AdS$_5$ black hole The near-horizon geometry of a macroscopic D3-brane is $AdS_5 \times S^5$. Once the brane is black we obtain the associated black hole solution with metric

$$ds^2 = \frac{r^2}{L^2}\left(-f(r)\ dt^2 + (d\vec{x})^2\right) + \frac{L^2}{r^2}\frac{dr^2}{f(r)} + L^2 d\Omega_5^2\,, \tag{5.1}$$

where $f(r) = 1 - (r_0/r)^4$ and RR field $C = C_{0\ldots 3} = [r^4/L^4 - r_0^4/2L^4]$. The constant part can be eventually absorbed into a redefinition of the parameter E.

By using eqs. (2.1) and (2.20), the effective density on a probe D3-brane in the above background turns out to be

$$\frac{8\pi}{3}\rho_{\rm eff} = \frac{1}{L^2}\left[\left(1+\frac{E}{a^4}\right)^2 - \left(1-\left(\frac{r_0}{L}\right)^4\frac{1}{a^4}\right)\left(1+\frac{\ell^2}{L^2}\frac{1}{a^6}\right)\right]. \tag{5.2}$$

When the brane is falling towards the black-brane the universe is contacting while if it moving outwards, it is expanding. Far from the black-brane $\rho_{\rm eff} \sim a^{-4}$. In this regime the brane motion produces a cosmological expansion indistinguishable with the one due to (dilute) radiation on the brane. As one goes backward in time there is a negative energy density $\sim a^{-6}$ controlled by the angular momentum ℓ. It corresponds to $\rho_{\rm eff} = p_{\rm eff}$, relation characteristic of a massless scalar. Although the density is negative the overall effective density remains non-negative for $0 < a < \infty$. At earlier times the factor $\sim a^{-8}$ dominates corresponding to dilute matter with $p = w\rho$ and $w = 5/3 > 1$. Such a behavior is unattainable by real matter on the brane since causality implies that $|w| \leq 1$. Finally, at very early times the evolution is dominated by mirage density with $w = 7/3$.

More generally we can consider the D3-brane moving in the background of a $p > 3$ black brane. This problem has been considered in a previous section. In this case we obtain an effective density using (2.32) with $p = 3$:

$$\frac{8\pi}{3}\rho_{\rm eff} = \left(\frac{7-p}{4L}\right)^2 a^{\frac{2(3-p)}{7-p}}\left[\frac{E^2}{a^{2(7-p)}} - \left(1-\left(\frac{r_0}{L}\right)^{7-p}\frac{1}{a^4}\right)\left(1+\frac{\ell^2}{L^2}\frac{1}{a^{2(5-p)+\frac{8}{7-p}}}\right)\right]. \tag{5.3}$$

As is obvious from the above, that the universe brane cannot go far away from the black-brane. It bounces back at some finite value of the scale factor. , this described a closed universe where the deceleration is provides by bulk fields rather than curvature on the brane. Particularly, interesting is the case $p = 5$ where (5.3) reads

$$\frac{8\pi}{3}\rho_{\rm eff} = \frac{1}{4L^2}\frac{1}{a^2}\left[\frac{E^2}{a^4} - \left(1-\left(\frac{r_0}{L}\right)^4\frac{1}{a^4}\right)\left(1+\frac{\ell^2}{L^2}\frac{1}{a^4}\right)\right]. \tag{5.4}$$

The term $-a^{-2}$ produces an effect similar to positive curvature and slows the expansion. The terms a^{-6} simulate the density of a massless scalar $w = 1$. This density increases with E and r_0 while the angular momentum tends to decrease the density. Moreover there is also a component with $w = 7/3$ In the case $\ell = r_0 = 0$, the universe expands until it eventually stops and eventually recolapses. The general picture with recolapse is also true for all $p > 3$. For $p = 4$ we obtain the effective indices $w = -7/9, 5/9, 11/9, 7/9, 19/9$. For $p = 6$ we obtain $w = 1, 5/3, 7/3, 3, 13/3$.

Dp-black brane The metric is

$$ds^2 = H_p^{-1/2}\left(-f dt^2 + (d\vec{x})^2\right) + H_p^{1/2}\left(\frac{dr^2}{f} + r^2 d\Omega_{8-p}^2\right), \tag{5.5}$$

with $H_p = 1 + L^{7-p}/r^{7-p}$, $f = 1 - r_0^{7-p}/r^{7-p}$. The RR field is $C = \xi(1 - H_p)/H_p$ with $\xi = \sqrt{1 + r_0^{7-p}/L^{7-p}}$ and the dilaton is $e^{\phi} = H_p^{(3-p)/4}$. A probe D3-brane in this background has a cosmological evolution driven by the effective density

$$\frac{8\pi}{3}\rho_{\text{eff}} = \frac{(1-a^4)^{5/2}}{L^2}\left[\frac{(E+\xi a^4)^2}{a^8} - \left(\xi^2 - \frac{\xi^2 - 1}{a^4}\right)\left(1 + \frac{\ell^2}{L^2}\frac{\sqrt{1-a^4}}{a^6}\right)\right], \tag{5.6}$$

when $p = 3$ and

$$\begin{aligned}\frac{8\pi}{3}\rho_{\text{eff}} = &\frac{(7-p)^2}{16L^2} a^{\frac{2(3-p)}{(7-p)}}(1-a^4)^{\frac{2(8-p)}{(7-p)}} \times \\ &\times\left[\frac{E^2}{a^{2(7-p)}} - \left(\xi^2 - \frac{\xi^2-1}{a^4}\right)\left(1 + \frac{\ell^2}{L^2}\frac{(1-a^4)^{\frac{2}{(7-p)}}}{a^{2(5-p)+\frac{8}{7-p}}}\right)\right],\end{aligned} \tag{5.7}$$

when $p > 3$

Here there is a limiting size for the scale factor namely $0 \leq a \leq 1$. the maximum value is in general a free parameter that can always be scaled to one in the equations. For $a \ll 1$ the cosmological evolution is similar to the one discussed for the near-horizon geometries. When $a \simeq 1$ there is a different type of evolution. Denoting $a^4 = 1 - \epsilon$, $\epsilon \ll 1$ we obtain (for $p < 7$)

$$\dot{\epsilon} = \frac{|7-p|}{L}\sqrt{E^2 - 1}\,\epsilon^{\frac{8-p}{7-p}} \qquad \Rightarrow \qquad \epsilon = \frac{L^{7-p}}{(\sqrt{E^2-1})^{7-p}} t^{-(7-p)}\,. \tag{5.8}$$

This is the typical behavior of asymptotically flat solutions. The case $p = 7, 8$ are not asymptotically flat. We will discuss $p = 8$ as an example below. The effective density is in this case

$$\frac{8\pi}{3}\rho_{\text{eff}} = \frac{1}{16L^2}a^{10}\left[E^2\, a^2 - \left(\xi^2 - \frac{\xi^2-1}{a^4}\right)\left(1 + \frac{\ell^2}{L^2}\frac{a^{14}}{(1-a^4)^2}\right)\right]. \tag{5.9}$$

For small a the dominant term in the evolution is a^6. For later times the scale factor will bounce depending on parameters before $a = 1$.

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As a conclusion for the asymptotic geometries of spherically symmetric branes, the evolution equation for the scale factor is different from standard evolution due to some kind of density on the brane. On the other it will be expected that once the universe brane is far away from the source and the gravitational and other fields are weak, in this regime the dominant source of cosmological expansion will be the matter density on the brane.

Near horizon of a D3-brane with a constant B-field The near-horizon geometry of a D3-brane with a constant B-field is still given by eq. (5.1). By using eq. (3.9) for $p = 3$, we find that the effective density on a probe D3-brane in the D3-brane background with a constant B-field turns out to be

$$\frac{4\pi}{3}\rho_{\rm eff} = \frac{1}{L^2}\left[\left(1+\frac{E}{a^4}\right)^2\left(1+\frac{b^2}{a^4}\right)^{-1} - \left(1-\frac{r_0^4}{L^4}\frac{1}{a^4}\right)\left(1+\frac{\ell^2}{L^2}\frac{\left(1+b^2/a^4\right)^{-1}}{a^6}\right)\right]. \tag{5.10}$$

Near horizon of a D3-brane with a world-volume electric field It is not difficult to see that the effect of the electric field affects the cosmological evolution as it would in a four dimensional universe.

In the case of the near-extremal D3-brane (5.1) we obtain instead of (5.2)

$$\frac{8\pi}{3}\rho_{\rm eff} = \frac{1}{L^2}\left[\left(1+\frac{E}{a^4}\right)^2\left(1+\frac{\mu^2}{a^4}\right) - \left(1-\left(\frac{r_0}{L}\right)^4\frac{1}{a^4}\right)\left(1+\frac{\ell^2}{L^2}\frac{1}{a^6}\right)\right]. \tag{5.11}$$

Thus, the extra electric field adds at late times an extra effective density $\Delta\rho{=}\mu^2{=}\mathcal{E}^2$.

This is a behavior that is expected: extra matter density on the brane, affects the geodesics (motion) , and this in turn affects the effective expansion of the brane universe. Moreover, this gives for small fields (or large scale factors) effects that are similar to those of a constant bulk NS antisymmetric tensor. Both produce an effect that can be interpreted as radiation in our brane universe.

6. The resolution of the initial singularity

In four dimensions standard cosmological models always carry an initial singularity. This is the point in the past of the evolution where the scale factor $a(t)$ goes to zero so that all space-like sections of space-time collapse to a point. This is a general feature and powerful theorems have established the occurrence of the initial singularity for matter obeying the energy-conditions [45]. Since the latter are satisfied for all known forms of matter, the initial singularity seems to be unavoidable. The basic assumption in the above is that the full description is given in terms of four-dimensional general relativity. If one views general relativity as an effective theory of a more fundamental theory, then the presence of the initial singularity may be resolved in the fundamental theory. One could argue that the singularity appears not

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because the fundamental solution is singular but because the effective field theory used to describe, is not valid in this regime [46, 47]. In particular, as discussed for example in [47], due to T-duality an initial singularity could really correspond to a decompactification limit for the relevant low energy (dual) modes.

In our context, there are cases where "mirage" energy violates the standard energy conditions. For example for dilute matter we have $p = w\ \rho$ with $|w| \leq 1$ for causality. In our the previous section we have found the mirage matter had $|w| > 1$ in most of the cases, (as well as some components of the density being negative) leading to the possibility of singularity-free evolution.

This can be seen in the simple example of the geodesic motion of a probe D3-brane in the near-horizon of macroscopic D5-branes with metric

$$ds^2 = \frac{r}{L}\left(-dt^2 + d\vec{x}_5^2\right) + \frac{L}{r}\left(dr^2 + r^2 d\Omega_3\right). \tag{6.1}$$

In this case, solving for $r(t)$ with $\ell = 0$ we get

$$r(t)^4 = L^4 E \cosh^{-2}\left(\frac{2t}{L}\right). \tag{6.2}$$

The induced metric (2.15) turns out to be

$$d\hat{s} = -\frac{E^{1/4}}{\cosh^{5/2}(2t/L)}dt^2 + \frac{E^{1/4}}{\cosh^{1/2}(2t/L)}d\vec{x}_3^2\,. \tag{6.3}$$

This four-dimensional metric has a singularity for $t = \infty$ (initial singularity). However, the higher dimensional geometry is regular. What becomes singular is the embedding of the brane in the bulk. Alternatively speaking, the four-dimensional singularity is smoothed out once the solution is lifted to higher dimensions. This is a well known method for desingularizing four-dimensional manifolds and appears here naturally.

Similarly, we may consider the motion of a probe D3-brane in $AdS_5 \times S^5$ background. Indeed we expect that for motion in this smooth manifold no real singularity on the brane can be encountered. Here we find from (2.12) and for $\ell = 0$ that $0 \leq r^4 < \infty$ for $E > 0$. The proper time η is then given

$$\eta = \frac{1}{2L}\sqrt{2r^4 + L^4} - \frac{L}{2}, \tag{6.4}$$

defined such that $\eta = 0$ for $r = 0$ and thus $0 \leq \eta < \infty$. On the other hand, solving for the scale factor $a(\eta)$ we find that

$$\alpha(\eta)^4 = \frac{8E}{L^2}\left((\eta + \eta_0)^2 - \frac{L^2}{16}\right), \tag{6.5}$$

where η_0 is an integration constant. At $\eta = 0$ we get that

$$\alpha(0)^4 = \frac{8E}{L^2}\left(\eta_0^2 - \frac{L^2}{16}\right), \tag{6.6}$$

so that we obtain the standard singularity for $|\eta_0| > L/4$. This is the point that the brane reaches $r = 0$ which is a coordinate singularity and otherwise a regular point of the AdS_5 space. Again, the embedding becomes singular there.

From the string theory picture we do understand that the initial singularity here corresponds to the probe brane coalescing with the other branes that generate the bulk background. The effective field theory on the brane is singular at this point because one has to take into account the non-abelian modes (zero length strings) that become massless. The interpretation of the real initial singularity here is as a breakdown of the effective low energy field theory description.

7. Incorporation into string theory

Orbifold or orientifold backgrounds in string theory have a brane interpretation. A fixed hyperplane of an orientifold transformation can be thought of as a bound state of an orientifold plane and a D-brane. The orientifold plane carries no degrees of freedom localized on it, but only some CP-odd couplings to cancel similar couplings of the bound D-brane. When moduli are varied, the D-brane(s) can move away from orientifold planes. In open string vacua, extra D-branes can participate in the structure of the vacuum.

In fact, a similar interpretation can be given to ordinary orbifold vacua of the heterotic and type-II strings.[1] The fixed planes of the orbifold are bound states of an orbifold plane and an NS-brane for a Z_2 twist. For a Z_N twist there are $N-1$ non-coincident NS5-branes. The degrees of freedom localized on the orbifold plane are essentially composed of the twisted sector fields of the orbifold. Turning on twisted moduli expectation values corresponds to moving the NS5-branes away from the orbifold plane.

Thus, a typical (orbifold) vacuum of type-I theory can be thought of as a collection of flat (toroidal) intersecting D-branes in a ten-dimensional flat bulk space. Constraints have to be satisfied, notably tadpole cancellation which ensures anomaly cancellation and reflects charge neutrality in a compact space. In the presence of some unbroken space-time supersymmetry, the configuration of D-branes is stable. Stability is a non-trivial requirement in the case of broken supersymmetry. It is however known that supersymmetric (BPS) D-branes have velocity dependent interactions [19]. Thus, there is non-trivial dynamics in the case that a D-brane is disturbed away from a stable configuration.

In this context, the Standard Model gauge bosons as well as charged matter arise as fluctuations of the D-branes. We can thus consider the universe (standard model) to be living on a collection of coincident branes, while hidden gauge interactions can be localized on other branes and gravity as well as other universal interactions is living in the bulk space.

[1]This is implicit in [44].

The approximation which we are using is very useful in order to treat the dynamics of the universe brane. This is the probe limit in which the influence of the probe brane source to the bulk fields is negligible. This has been a natural and useful tool [20]–[22] in order to understand issues in the context of AdS/CFT correspondence [23]. However, if we can approximate the collection of other branes (except the universe brane) that form the string vacuum as a spherically symmetric configuration, then our treatment is (classically) exact. The reason is the back-reaction to the bulk metric due to the probe brane modifies the bulk metric for radii larger that the position of the brane.

8. Conclusions and further directions

We have pointed out that if our universe is a D-brane embedded in a higher dimensional bulk, the motion of the brane in nontrivial bulk backgrounds with a certain symmetry (spherical in our case) produces a homogeneous and isotropic cosmological evolution on the universe brane. By considering various bulk background fields, we have derived Friedman-like equations. These provide cosmological evolution that can be attributed to matter density on the universe brane, although they are due to motion in nontrivial bulk fields. From the universe brane perspective such energy density is a mirage. The only way to tell whether the energy density driving the evolution of the universe is in the universe itself is by direct observation with conventional means (light for example).

It is also important to point out that we consider a situation where the spatial sections of the universe brane are flat. Unlike the typical four-dimensional case though, the effects of non-trivial spatial curvature (a a^{-2} term in the Friedman equation) can be simulated by the brane motion. We have seen in section 5 that a D3-brane moving the field of parallel D5-branes produces an effect similar to having a negative constant curvature in the spatial slice.

We should also point out here that it is also possible to consider situations where the induced brane metric really does have positive constant curvature. That should be produced by a bulk $B_{\mu\nu}$ field with $H_{\mu\nu\rho} \sim \epsilon_{\mu\nu\rho}$. Finding the precise solution in IIA,B supergravity is an interesting problem. such a solution though is known in gauged-supergravity in five-dimensions. The only difference in our equations is to replace the spacial section metric on the brane by a constant curvature one, and add the usual curvature contribution $-k/a^2$ to our Friedman-like equations.

When we have extra world-volume fields excited on the brane, these affect the universe brane motion, the induced metric and thus the cosmological evolution. We have analysed the case of electromagnetic energy (electric fields) and shown that it affects the cosmological evolution as normal radiation would. This indicates that the present formalism is capable of describing the most general cosmological evolution on a universe brane. This is driven by bulk fields (that we could label “bulk energy”)

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as well as world-volume fields ("brane-energy"). The cosmological evolution in this context has the simple and appealing interpretation of brane motion in the bulk.

Generically, the Friedman-like equations we find, contain components that can be interpreted as dilute matter with $|w| > 1$ that would otherwise clash with causality in four-dimensions. This may be an indication that the bulk can support superluminal signal propagation from the brane point of view. Moreover, sometimes, some effective density coefficients can be negative (without spoiling the positivity of the overall effective density). This violates typical four-dimensional positive energy conditions.

A Friedman type cosmological evolution is equivalent to a variable speed of light in the context of non-dynamical gravity. Cosmological implications of a variable light speed have been recently discussed in [48]. Our description here can be cast in that language [22]. In [22] it was pointed out that a natural way of inducing a field-dependent light velocity of brane is in the presence of a (cosmological) black brane. Subsequent motion of the probe brane, either falling into the black brane, or escaping outwards (a Hawking radiated brane?) would make the velocity of light field dependent and thus induce cosmological evolution.

The issue of the initial singularity has a natural resolution in this context. Although the universe brane has a singular geometry at zero scale factor, this is due to a singular embedding in the otherwise regular bulk space. This resolution of singularities in higher dimensions is not new but occurs naturally here. Relativistic corrections are important when one approaches the initial singularity. They invalidate for example naive treatments of brane fields when the universe brane approaches other branes.

There are some interesting open problems in this line of thought.

- The first concerns the possibility of inducing and ending inflation as a consequence of brane motion. This is similar in spirit with [24] but the context is different: here inflation is not generated by four-dimensional gravity on the brane. This will provide for a natural source of inflation since the inflaton is supplied automatically once we decide that our four-dimensional universe is a soliton-like object in a higher-dimensional theory.

- A second direction is finding bulk configurations that induce a cosmological evolution on the brane similar to that of dust ($w = 0$). Such mirage matter might be a component of dark matter on our universe. It is conceivable that such "mirage" matter could gravitate and trace the visible (world-volume) matter as observation suggests.

- Finding brane configurations in ten-dimensional supergravity with non-zero spatial curvature is also an interesting problem.

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Note added While this work was written up, reference [49] appeared where a similar problem is tackled with a different technique (namely using the Israel matching conditions). The resulting equation (17) for brane motion in an AdS black-hole (with $\sigma = \sigma_c, k = 0$) matches our equation (5.2) with $\ell = 0$ with the identifications $E = \mu_+ - \mu_-/2L^2$, $(r_0/L)^4 = 2\mu_-/l^2$, $L = l$. This is as expected since the energy of motion E creates a discontinuity of the black-hole horizon position μ in the bulk. Here we have a direct D-brane description which requires $\sigma = \sigma_c$.

We also became aware of the work in ref. [50] which partly overlaps with our work. There, the Israel matching conditions are also used to study brane motion and some interesting exact solutions are found, exhibiting cosmological evolution.

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